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|---|---|
| **Part B** | Problems 1-10 which only require answers. |
| **Part C** | Problems 11-15 which require complete solutions. |
| **Test time** | 120 minutes for Part B and Part C together. |
| **Resources** | Formula sheet and ruler. |

**Level requirements**

The test consists of an oral part (Part A) and three written parts (Part B, Part C and Part D). Together they give a total of 64 points consisting of 23 E-, 22 C- and 19 A-points.

Level requirements for test grades
E: 16 points
D: 25 points of which 7 points on at least C-level
C: 33 points of which 13 points on at least C-level
B: 43 points of which 6 points on A-level
A: 52 points of which 11 points on A-level

The number of points you can have for a complete solution is stated after each problem. You can also see what knowledge level(s) (E, C and A) you can show in each problem. For example (3/2/1) means that a correct solution gives 3 E-, 2 C- and 1 A-point.

For problems labelled "*Only answers required*" you only have to give a short answer. For other problems you are required to present your solutions, explain and justify your train of thought and, where necessary, draw figures.

**Write your name, date of birth and educational programme on all the sheets you hand in.**

Name: ______________________________

Date of birth: ______________________________

Educational programme: ______________________________

**Part B:** Digital resources are not allowed. *Only answer is required*. Write your answers in the test booklet.

**1.** It holds for the polynomial function $f$ that $f(x) = 3x^4 + 7x^2 + 3$

a) What is the degree of the function $f$? _______________ (1/0/0)

b) Determine $f'(x)$. _______________ (1/0/0)

**2.** Specify two different antiderivatives of $f(x) = 7x + 4$

_______________ and _______________ (2/0/0)

**3.** During the first seconds after starting the distance a car travels can be described with $s(t) = 3t + t^2$ where *s* is the distance in metres and *t* is the time in seconds.

Determine the speed *v* of the car as a function of time *t*.

$v(t) =$ _______________ (1/0/0)

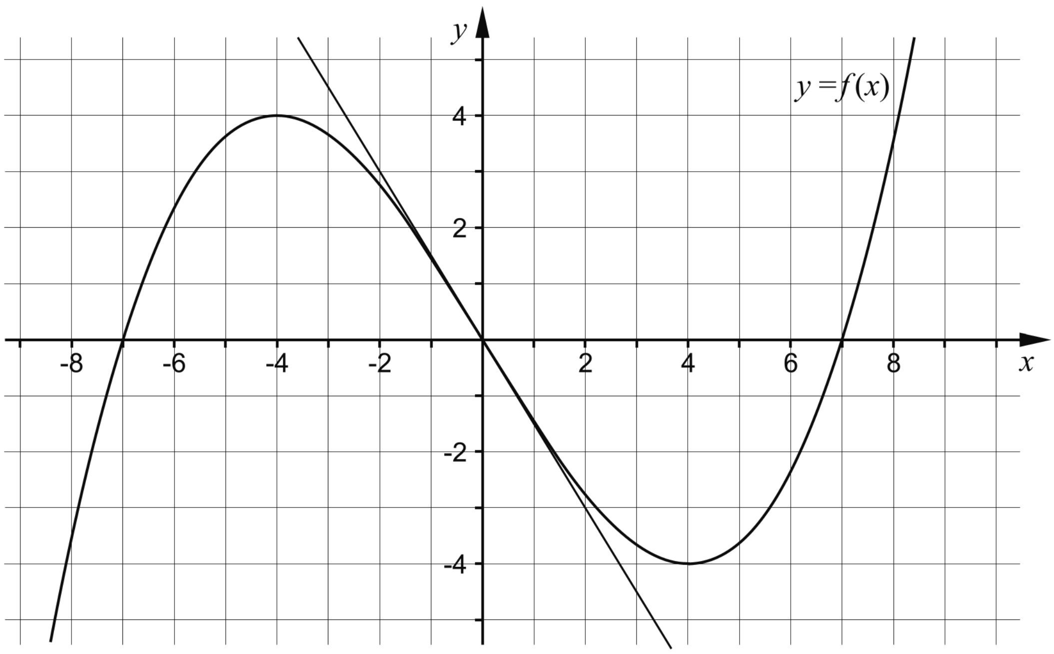
**4.** Solve the equation $(x+2)(x-3)(x+4) = 0$ _______________ (1/0/0)

**5.** Simplify the following expressions as far as possible.

a) $16 + (x^3 + 4)(x^3 - 4)$ _______________ (1/0/0)

b) $\dfrac{x}{(x+4)^9} + \dfrac{4}{(x+4)^9}$ _______________ (0/1/0)

**6.** The figure below shows the graph of a cubic function $f$ and a tangent which touches the graph at the origin.




a) Determine the zeroes of the derivative. ______________________ (1/0/0)

b) Determine $f'(0)$. ______________________ (1/0/0)

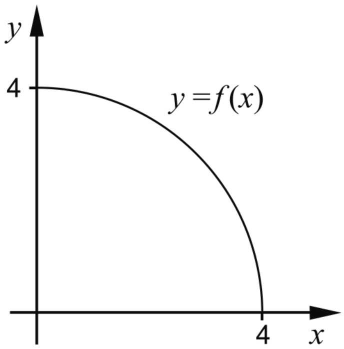
c) Sketch the graph of the derivative of the function in the coordinate system above. (0/1/1)

**7.** The geometric sum $2-2\cdot 1.5+2\cdot 1.5^2-2\cdot 1.5^3+...-2\cdot 1.5^{19}$ equals one of the alternatives A-H below. Which?

| | | | |
|---|---|---|---|
| A. $2\cdot\dfrac{(-1.5)^{18}-1}{-1.5-1}$ | B. $2\cdot\dfrac{(-1.5)^{19}-1}{-1.5-1}$ | C. $2\cdot\dfrac{(-1.5)^{20}-1}{-1.5-1}$ | D. $2\cdot\dfrac{(-1.5)^{21}-1}{-1.5-1}$ |
| E. $2\cdot\dfrac{1.5^{18}-1}{1.5-1}$ | F. $2\cdot\dfrac{1.5^{19}-1}{1.5-1}$ | G. $2\cdot\dfrac{1.5^{20}-1}{1.5-1}$ | H. $2\cdot\dfrac{1.5^{21}-1}{1.5-1}$ |

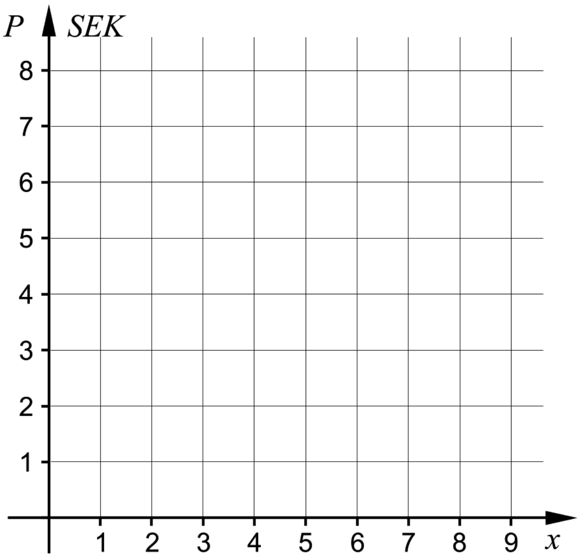
______________________ (0/1/0)

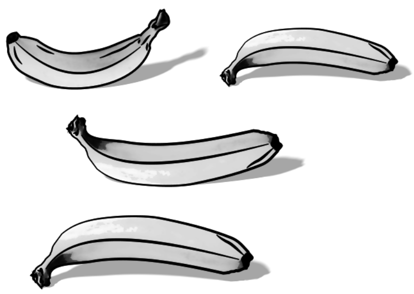
**8.** The graph of the function $f$ forms a quarter circle in the first quadrant.




Evaluate $\int_0^4 f(x)\,dx$. Give an exact answer. ______________________ (0/1/0)

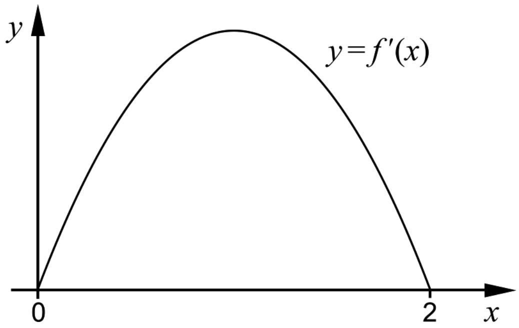
**9.** The price of bananas at the cafeteria of Hagaskolan is SEK 2 a piece. The price SEK $P$ is a function of the number of bananas $x$. Draw the graph of the function within the interval $1 \leq x \leq 4$ in the coordinate system below.



(0/1/0)

**10.** The function $f$ has the derivative $f'$. The figure below shows the graph of $f'$. Which of the statements A-F is *always* true?




A. $f(2)$ is positive

B. $f(2) - f(0)$ is positive

C. $f(1)$ is zero

D. $f(0)$ is zero

E. $f(1) - f(2)$ is positive

F. $f(0) - f(1)$ is positive ______________________ (0/0/1)

**Part C**: Digital resources are not allowed. Do your solutions on separate sheets of paper.

**11.** The figures A and B below show the graphs of two cubic functions.

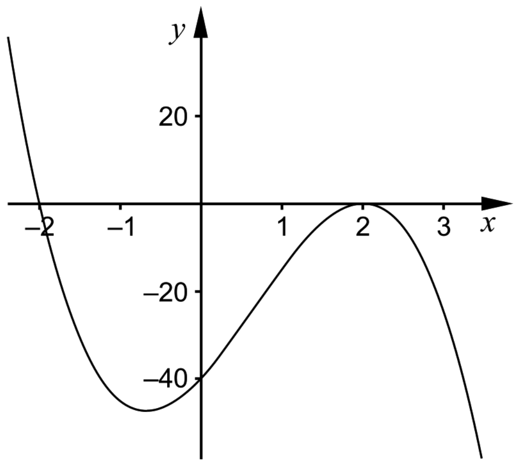



Figure A

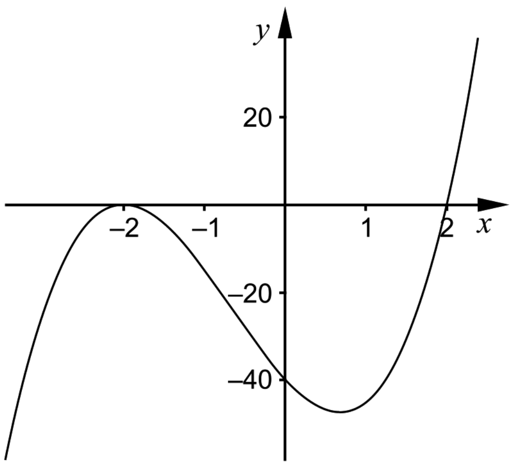


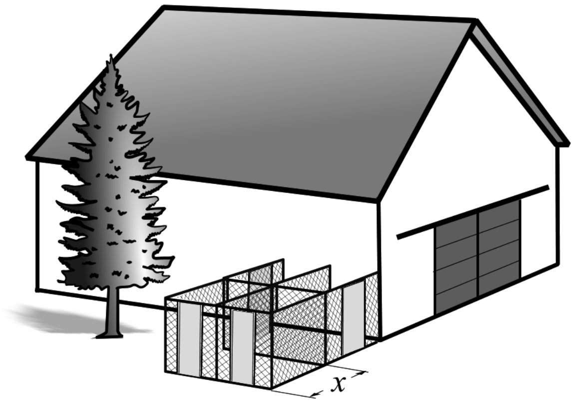

Figure B

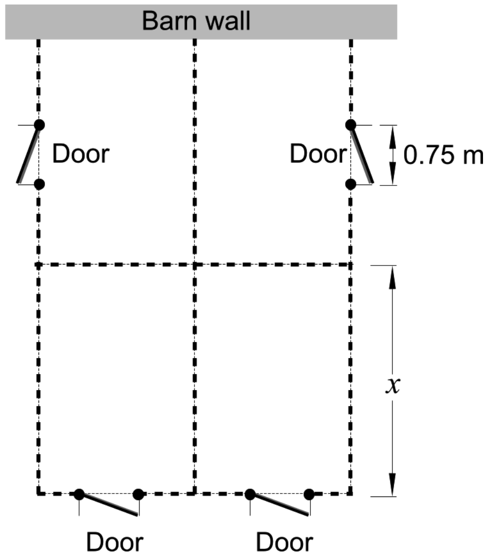
a) Which of the figures shows the graph of a cubic function $f$ where $f'(2)=0$? Justify your answer. (1/0/0)

b) Which of the figures shows the graph of $f(x)=5(x-2)(x+2)^2$? Justify your answer. (0/1/0)

**12.** Karin is going to build four rectangular shaped dog runs for her dogs. All four dog runs will have the same dimensions and will be enclosed by fencing.

Karin has 45 m of fencing and four doors that she will use for the dog runs. Two of the dog runs will be built against a barn wall. Therefore, fencing will not be needed on the side made by the barn wall. The doors are 0.75 m wide, with the same height as the fencing and will be placed according to the figure.




The area of each of the dog runs is given by the function $A(x)=12x-1.5x^2$ where $A$ is the area in $m^2$ and $x$ is the length in m of one of the sides of the dog run, se figure.

a) Use the derivative to find the value of $x$ that gives the largest possible area for each dog run. (2/0/0)

b) Show that the area of a dog run is given by the function $A(x)=12x-1.5x^2$ (0/0/3)

**13.** Solve the equation $\dfrac{6}{x-3}-\dfrac{18}{x(x-3)}=2$ (0/3/0)

**14.** Evaluate $\displaystyle\int_0^4 e^{\frac{x}{2}}\,dx$. Give an exact answer. (0/2/0)

**15.** Determine the constant $A$ so that $\displaystyle\lim_{x\to\infty}\frac{Ax}{4x+A}=\frac{1}{7}$ (0/0/3)