Part B	Problems 1–11 which only require answers.
Part C	Problems 12–17 which require complete solutions.
Test time	120 minutes for Part B and Part C together.
Resources	Formula sheet and ruler.

Level requirements

The test consists of an oral part (Part A) and three written parts (Part B, Part C and Part D). Together they give a total of 68 points consisting of 24 E-, 25 C- and 19 A-points.

Level requirements for test grades
E: 18 points
D: 27 points of which 8 points on at least C-level
C: 35 points of which 14 points on at least C-level
B: 45 points of which 6 points on A-level
A: 54 points of which 11 points on A-level

The number of points you can get for a complete solution is stated after each problem. You can also see what knowledge levels (E, C and A) you can show in each problem. For example (3/2/1) means that a correct solution gives 3 E-, 2 C- and 1 A-point.

For problems labelled "*Only answer is required* " you only have to give a short answer. For other problems you are required to present your solutions, explain and justify your train of thought and, where necessary, draw figures.

Write your name, date of birth and educational programme on all the sheets you hand in.

Name: _____

Date of birth: _____

Educational programme: _____

Part B: Digital resources are not allowed. *Only answer is required.* Write your answers in the test booklet.

1. Determine the degree of the polynomial $x^5 + 7x^4 + 3x - 8$ _____ (1/0/0)

2. Avasjö-Borgafjäll is a weather observation station where SMHI collects weather data. The diagram shows how the snow depth (in cm) varies over one month, starting April 1, 2013 at 07.00.

Avasjö-Borgafjäll, snow depth April 2013
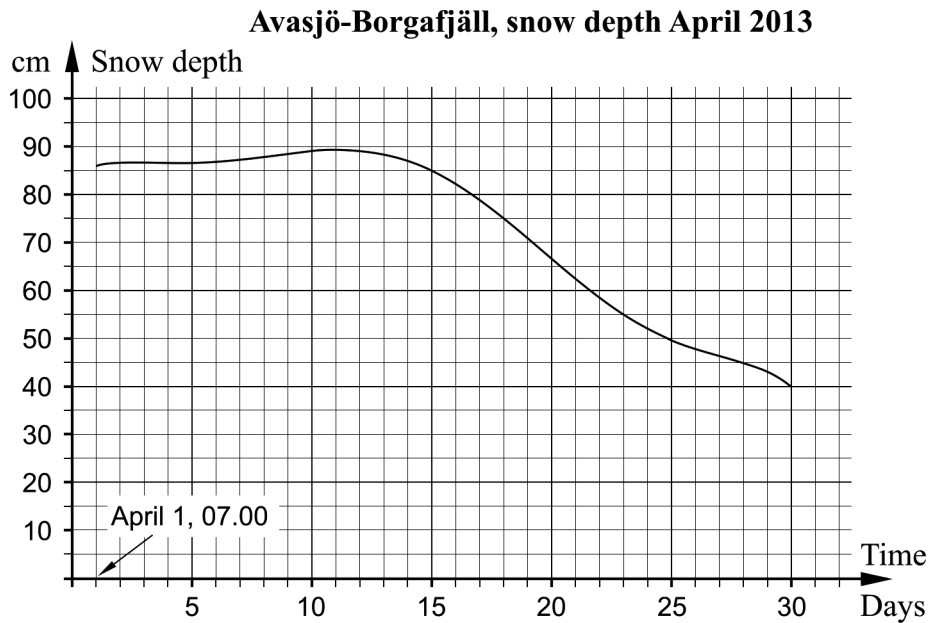
Answer the following questions by using the diagram.

a) At what average rate of change in cm/day does
the snow depth decrease over the period
April 15, 07.00 to April 30, 07.00? _____ (1/0/0)

b) Determine the rate of change in cm/day for
the snow depth on April 11, 07.00. _____ (1/0/0)

c) When does the snow depth decrease the fastest?
Choose one of the alternatives A–E.

    A.   April 1
    B.   April 11
    C.   April 14
    D.   April 20
    E.   April 28 _____ (1/0/0)

3. For what value of x is the expression $\dfrac{2x+10}{3x-6}$

not defined? _____ (1/0/0)
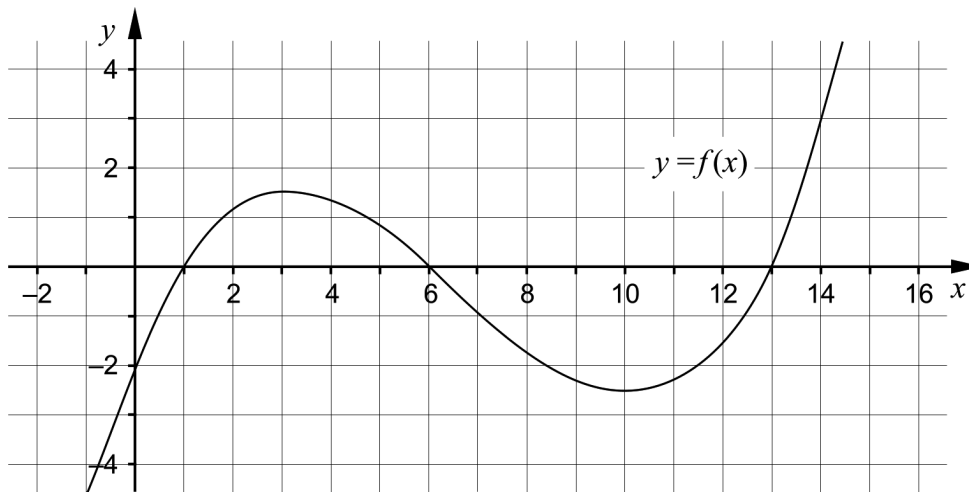
4. Simplify $x^2(x^3+9)-(3x)^2$ as far as possible. _____ (1/0/0)

5. Find $f'(x)$ if

a) $f(x)=3x^4+x^2-5$ $f'(x)=$ _____ (1/0/0)
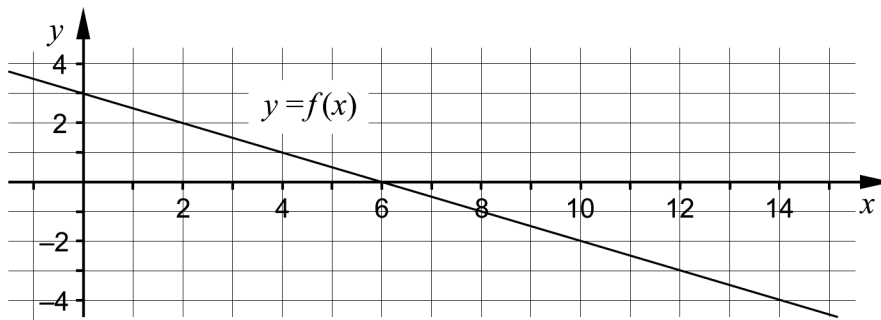
b) $f(x)=\dfrac{kx-1}{4}$ $f'(x)=$ _____ (0/1/0)

6. The figure shows the graph of the cubic function f.

a) Use the figure to find the zeroes of the derivative.

_____ (1/0/0)

b) It holds for another function g that $g(x)=-f(x)$.
Mark the points $A=(1,g(1))$, $B=(3,g(3))$ and $C=(14,g(14))$
in the figure. (0/1/0)

3

7. The graph of the function f is a straight line, see figure.

Find the upper bound a where $a \neq 0$

so that $\displaystyle\int_0^a f(x)\,dx = 0$ _____ (0/1/0)

8. The function f describes the number of inhabitants in a municipality as a function of time t, where t is the time in years after January 1, 2013.

   There are four empty boxes in the figure. Write suitable numbers and symbols in the boxes so the interpretation of the equality is:

   During the period January 1 2015 to January 1 2020, the number of inhabitants increases by 45 107.

$$\int_{\boxed{}}^{\boxed{}} \boxed{}\,dt = \boxed{}$$

(0/1/1)

9. Determine $\displaystyle\lim_{x \to 0}\frac{\ln e^x + xe^x}{5x}$ _____ (0/0/1)

10. Find the roots of the equation
    $$\frac{(x-5)^4 - (x-5)^3}{x-10} = 0$$

_____ (0/0/1)

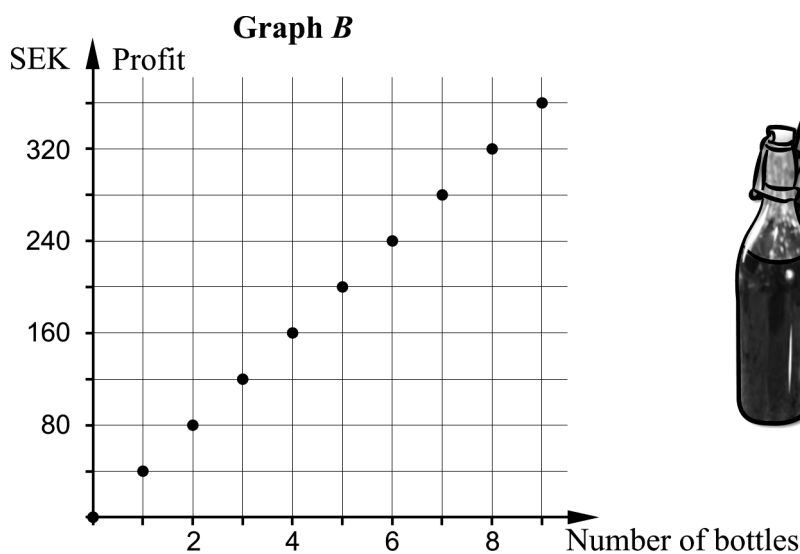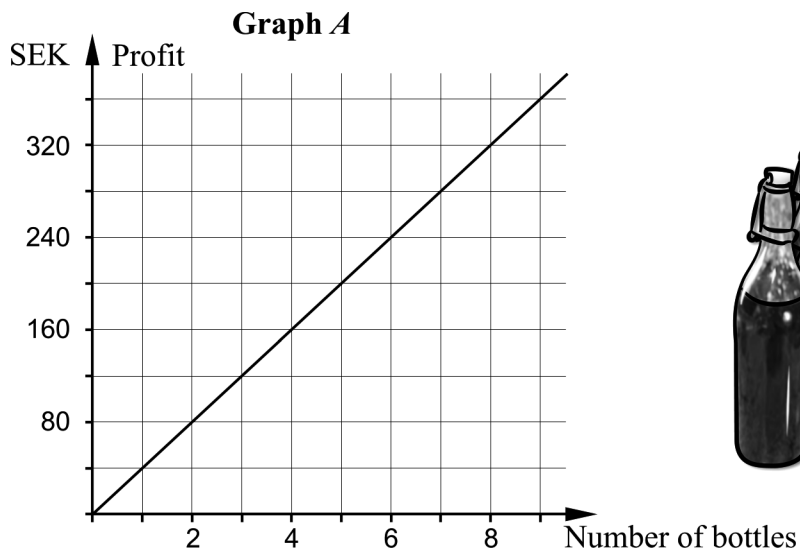11. The derivative of $f(x) = \sqrt{\pi} \cdot \sqrt{x}$ can be written in

    the form $f'(x) = \sqrt{\dfrac{\pi}{ax}}$

    Determine the constant a. _____ (0/0/1)

Part C: Digital resources are not allowed. Write down your solutions on separate sheets of paper.

12. Daniel and Jakob sell bilberry lemonade. The price of each bottle is SEK 40. Which of the graphs, *A* or *B*, best describes the profit of the lemonade sale? Justify your answer. (1/0/0)

Graph *A*

Graph *B*

13. It holds for the function f that $f(x) = x^3 - 12x$

Use the derivative to determine the coordinates for any maximum points, minimum points and saddle points for the graph to the function.

Determine also the characteristics for each point, that is whether it is a maximum point, minimum point or saddle point. (3/1/0)
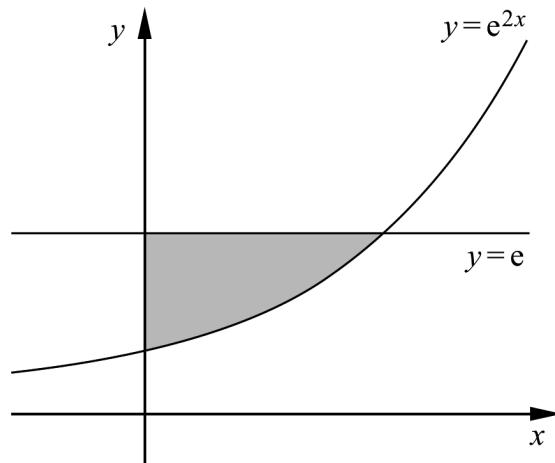
14. Evaluate the integrals algebraically.

a) $\displaystyle\int_{0}^{3} (x^2 - 2)\, dx$ (2/0/0)

b) $\displaystyle\int_{1}^{2} (x + \frac{1}{x^3})\, dx$ (0/2/0)

15. Solve the equation $\dfrac{3}{9 - 3x} = \dfrac{1}{3} - \dfrac{2}{3 - x}$ (0/2/0)
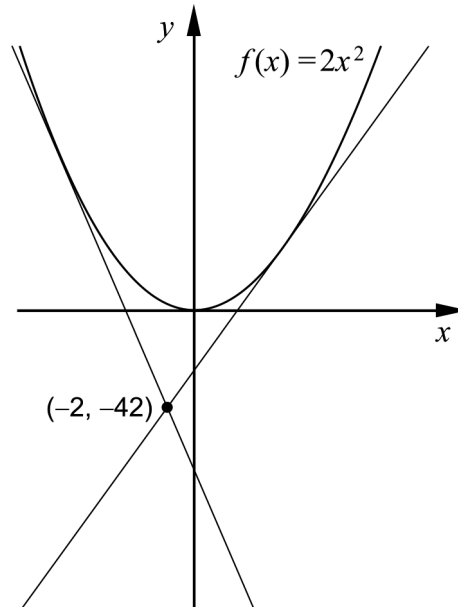
16. The grey shaded region in the figure is bounded by the curve $y = e^{2x}$, the line $y = e$ and the positive y-axis.

Calculate the area of the grey shaded region algebraically and give your answer in the simplest possible form. (0/3/0)

17. It holds for the function f that $f(x) = 2x^2$. The graph of the function has two tangents that pass through the point $(-2, -42)$, see figure.

Find the equation for one of the two tangents algebraically. (0/0/4)