Part B	Problems 1–11 which only require answers.
Part C	Problems 12–17 which require complete solutions.
Test time	120 minutes for Part B and Part C together.
Resources	Formula sheet and ruler.

The test consists of an oral part (Part A) and three written parts (Part B, Part C and Part D). Together they give a total of 67 points consisting of 24 E-, 24 C- and 19 A-points.

Level requirements for test grades
E: 17 points
D: 26 points of which 8 points on at least C-level
C: 34 points of which 14 points on at least C-level
B: 44 points of which 6 points on A-level
A: 53 points of which 11 points on A-level

The number of points you can get for a complete solution is stated after each problem. You can also see what knowledge levels (E, C and A) you can show in each problem. For example (3/2/1) means that a correct solution gives 3 E-, 2 C- and 1 A-point.

For problems labelled "*Only answer is required*" you only have to give a short answer. For other problems you are required to present your solutions, explain and justify your train of thought and, where necessary, draw figures.

Write your name, date of birth and educational programme on all the sheets you hand in.

Name: _____

Date of birth: _____

Educational programme: _____

Part B: Digital resources are not allowed. *Only answer is required*. Write your answers in the test booklet.

1. Find *all* antiderivatives of $f(x) = x^2 + 8$

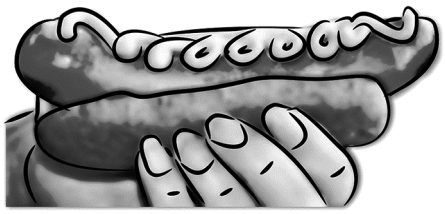
$F(x) =$ _____ (1/0/0)

2. Ayse throws a ball straight up into the air. The height of the ball is given by $s(t) = 1.5 + 12t - 5t^2$
where $s(t)$ is the height above the ground in metres and t is the time in seconds after the throw.

Determine the velocity of the ball at the time $t = 1$ second.

_____ m/s (1/0/0)

3. Mattias sells hot dogs at a football game. The hot dogs cost SEK 20 each. Mattias' revenue in SEK is a function of the number of hot dogs sold.



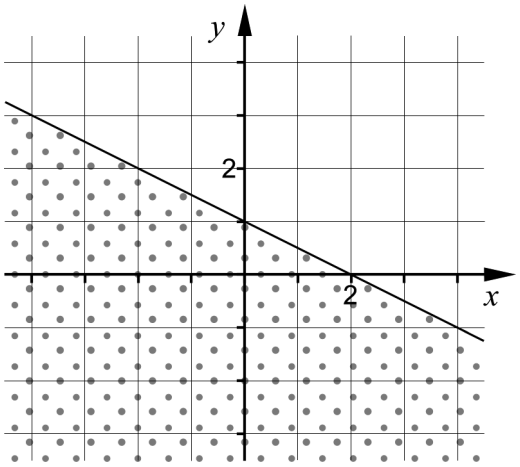
Which of the alternatives A–E is correct?

The function is …

A. a quadratic function.
B. a discrete function.
C. an exponential function.
D. a constant function.
E. a continuous function.

_____ (1/0/0)

4. One of the alternatives A–F corresponds to the dotted area.
Which one? _____ (1/0/0)



A. $y+0.5x \geq 1$ D. $y-0.5x \geq 1$

B. $y+0.5x \leq 1$ E. $y-0.5x \leq 1$

C. $y+0.5x = 1$ F. $y-0.5x = 1$

5. Determine $f'(x)$.

a) $f(x) = 5x^5 + x^2 - 2$ $f'(x) =$ _____ (1/0/0)

b) $f(x) = \dfrac{e^{4x} - e}{3}$ $f'(x) =$ _____ (0/1/0)

c) $f(x) = -\dfrac{2}{\sqrt{x}}$ $f'(x) =$ _____ (0/1/0)

6. In the expression $\dfrac{x-A}{2B-x^2}$, A and B are constants.

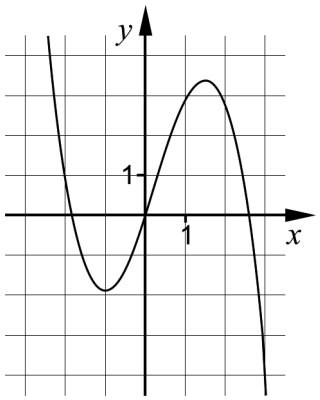
Determine A and B so that the following two conditions are satisfied:

- The expression has the value 0 when $x = -5$
- The expression is not defined for $x = 10$ and $x = -10$

$A =$ _____ (0/1/0)

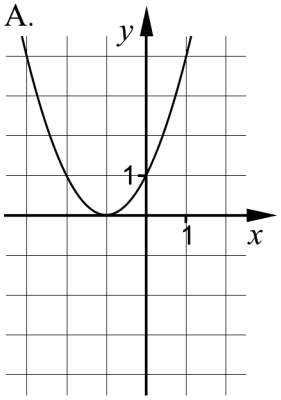
$B =$ _____ (0/1/0)

7. The figure shows the graph of the function f.

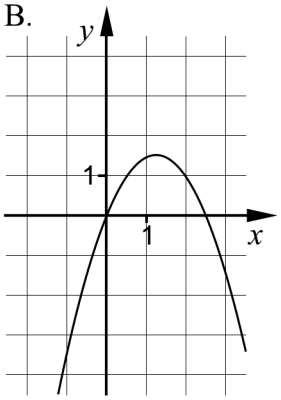


One of the alternatives A–F shows the graph of the function's derivative f'. Which one? _____ (0/1/0)

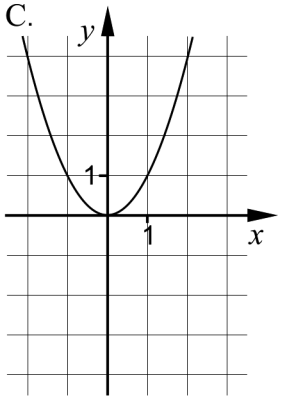
A.



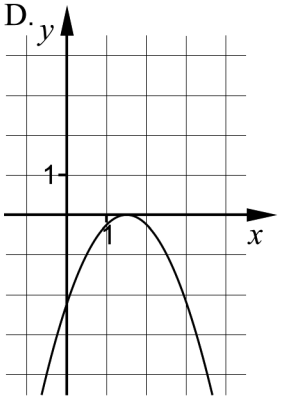
B.



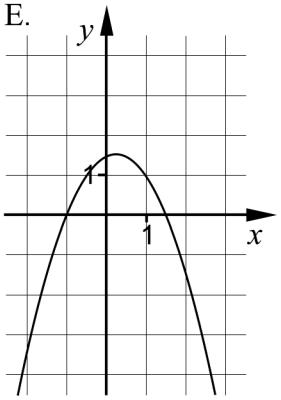
C.



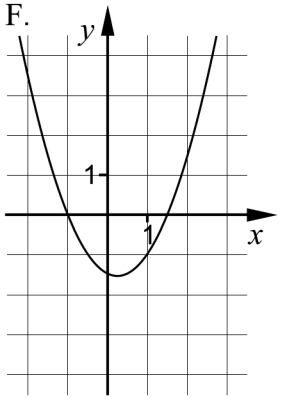
D.



E.



F.



8. Simplify the expressions as far as possible.

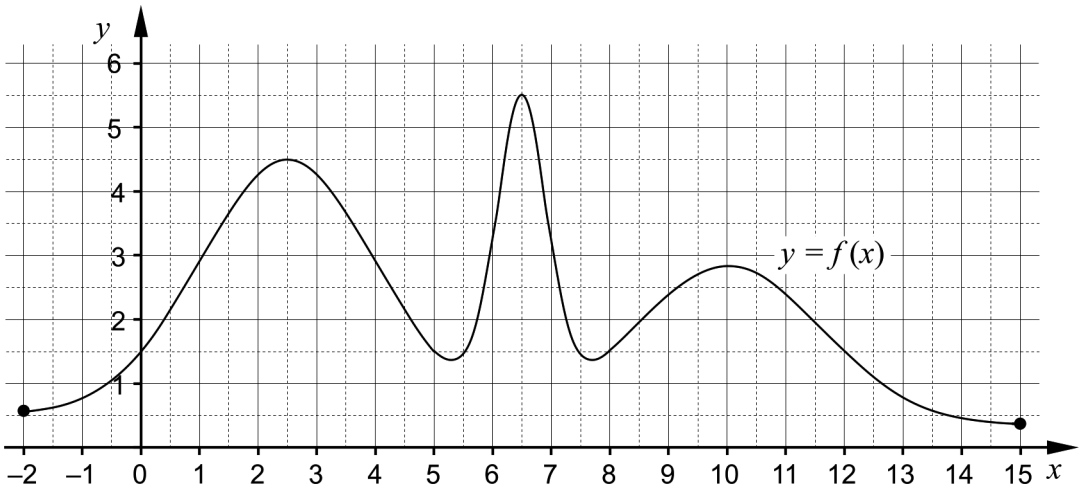
a) $\dfrac{2x-10}{2x^2-20x+50}$ _____ (0/1/0)

b) $-x^4-(-2x)^4$ _____ (0/1/0)

c) $\dfrac{-A+(A+5)^{10}-5}{A+5}$ _____ (0/0/1)

9. The first four numbers in a geometric progression are
$1,\ a_2,\ a_3,\ 64$
Determine a_2 _____ (0/1/0)

10. The figure shows the graph of the function f on the interval $-2 \le x \le 15$



For what value of p does $\int_{p}^{p+2} f(x)\,\mathrm{d}x$ assume its largest value?

_____ (0/0/1)

11. The function g is a cubic function. The table shows the sign of the derivative for some different values of x.

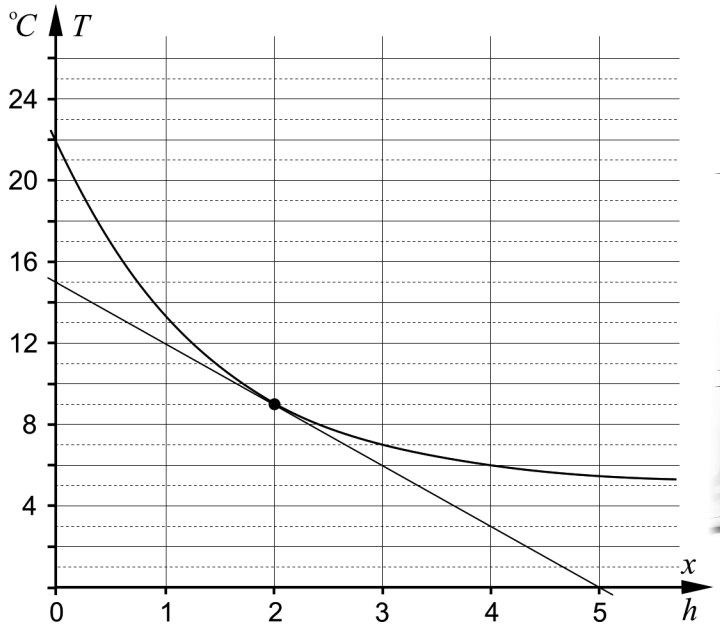
x	0	3	5	6	10
$g'(x)$	–	0	+	0	–

For what value of x does it hold that $g''(x)=0$? _____ (0/0/1)

Part C: Digital resources are not allowed. Write down your solutions on separate sheets of paper.

12. Evaluate $\int_1^2 6x^2 \mathrm{d}x$ algebraically. (2/0/0)

13. A bottle of water is placed in a fridge at 12:00. The temperature of the water is described by the exponential function $T(x) = 17\mathrm{e}^{-0.7x} + 5$ where $T(x)$ is the temperature of the water in $°C$ and x is the time in hours after 12:00. The figure shows the graph of the function T and the tangent at the point where $x = 2$



a) Read from the figure and calculate the average temperature change of the water per hour during the first 4 hours. (2/0/0)

b) Use the figure to calculate the gradient of the tangent. Interpret the meaning of this gradient in this context. (0/2/0)

c) Is it possible for the temperature of the water to reach $3°C$?
Start with the exponential function $T(x) = 17\mathrm{e}^{-0.7x} + 5$ and justify your answer. (1/1/0)

14. It holds for the function f that $f(x) = x^3 - 6x^2 + 9x$
Use the derivative to determine the coordinates for any existing maximum, minimum and saddle points for the graph of the function.

Also, determine the character of each point, that is, whether it is a maximum, minimum or saddle point. (3/1/0)

15. Maja investigates the cubic equation $(2x-1)(x^2+4) = 0$ and claims:

"The equation has three real solutions."

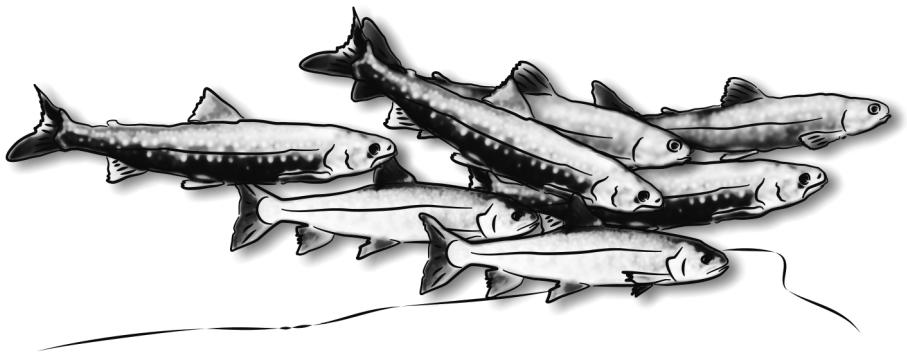
Investigate whether she is right. (2/0/0)

16. It holds for a function f that $f(x) = kx + m$

Investigate what must be true for k and m if $\int_{-2}^{2} f(x)\,\mathrm{d}x = 4$

Justify your conclusions. (0/2/1)

17. Fish of a species that has not existed there before are planted into a lake. The population of fish can be described by the relation $N(t) = \dfrac{15\,000}{3 + 2\mathrm{e}^{-0.5 \cdot t}}$ where N is the number of fish and t is the time in years after the fish have been planted.



a) Determine how many fish were initially planted into the lake. (0/1/0)

b) Due to different environmental factors, the number of fish cannot keep growing indefinitely. Use the relation and determine the upper limit for the number of fish. (0/0/2)