

<b>Part B</b>	Problems 1–8 which only require answers.
<b>Part C</b>	Problems 9–18 which require complete solutions.
<b>Test time</b>	150 minutes for Part B and Part C together.
<b>Resources</b>	Formula sheet and ruler.

### Level requirements

The test consists of three written parts (Part B, Part C and Part D). Together they give a total of 61 points consisting of 21 E-, 23 C- and 17 A-points.

Level requirements for test grades

E: 15 points

D: 24 points of which 7 points on at least C-level

C: 31 points of which 13 points on at least C-level

B: 41 points of which 5 points on A-level

A: 49 points of which 9 points on A-level

The number of points you can get for a complete solution is stated after each problem. You can also see what knowledge levels (E, C and A) you can show in each problem. For example (3/2/1) means that a correct solution gives 3 E-, 2 C- and 1 A-point.

For problems labelled “*Only answer is required*” you only have to give a short answer. For other problems you are required to present your solutions, explain and justify your train of thought and, where necessary, draw figures.

**Write your name, date of birth and educational programme on all the sheets you hand in.**

Name: \_\_\_\_\_

Date of birth: \_\_\_\_\_

Educational programme: \_\_\_\_\_

**Part B:** Digital resources are not allowed. *Only answer is required.* Write your answers in the test booklet.

1. It holds for the function  $f$  that  $f(x) = \sin 2x$ .

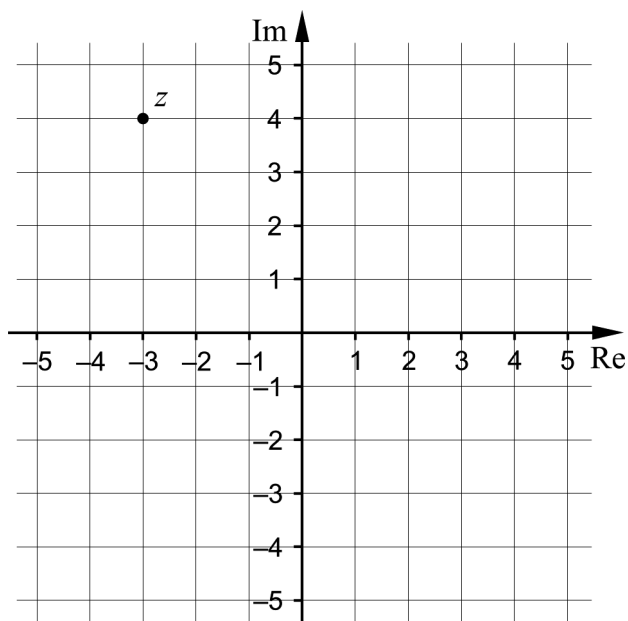
a) Find  $f\left(\frac{\pi}{6}\right)$ . \_\_\_\_\_ (1/0/0)

b) Find  $f'(x)$ . \_\_\_\_\_ (1/0/0)

2. Write down the vertical asymptotes of  $f(x) = \frac{1}{x^2 - 4}$

\_\_\_\_\_ (2/0/0)

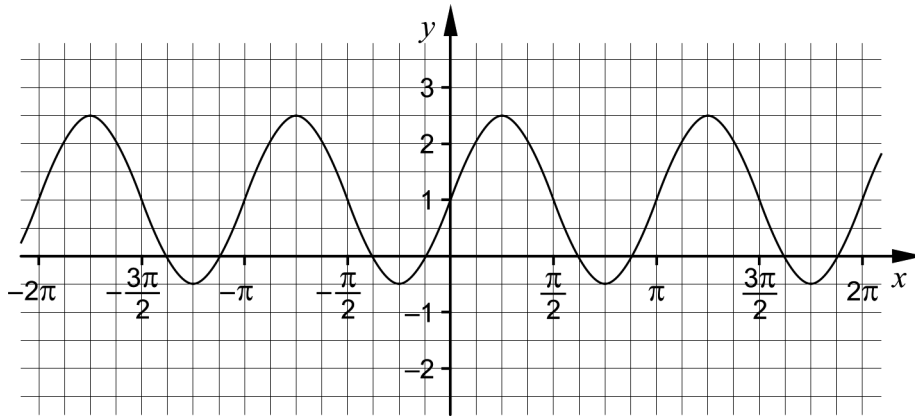
3. The figure shows a complex plane where the number  $z$  has been marked.



a) Find  $\bar{z}$ . \_\_\_\_\_ (1/0/0)

b) Find  $|z|$ . \_\_\_\_\_ (1/0/0)

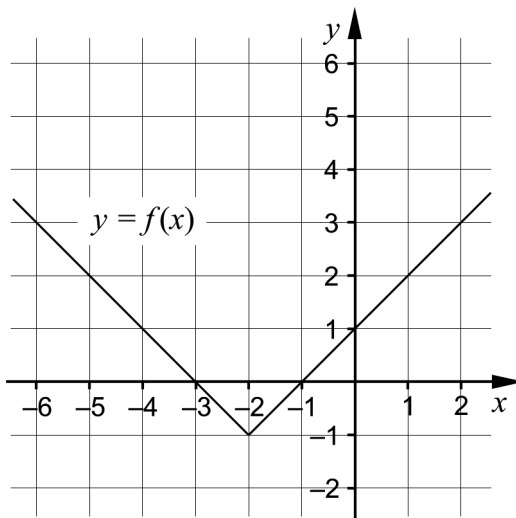
4. The figure shows a sine curve.



Find the equation of the sine curve in the form  $y = A \sin(kx) + B$ .

$y =$  \_\_\_\_\_ (1/1/0)

5. The figure shows the graph of  $f(x) = a + |x + b|$ .

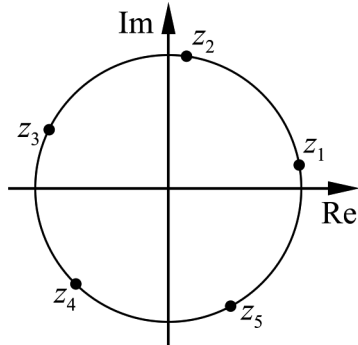


Find the constants  $a$  and  $b$ .

$a =$  \_\_\_\_\_

$b =$  \_\_\_\_\_ (1/1/0)

6. The figure shows the circle  $|z|=1$  in the complex plane. The five roots  $z_1, z_2, z_3, z_4$  and  $z_5$  of the equation  $z^5 = \cos 50^\circ + i \sin 50^\circ$  are marked on the circle.



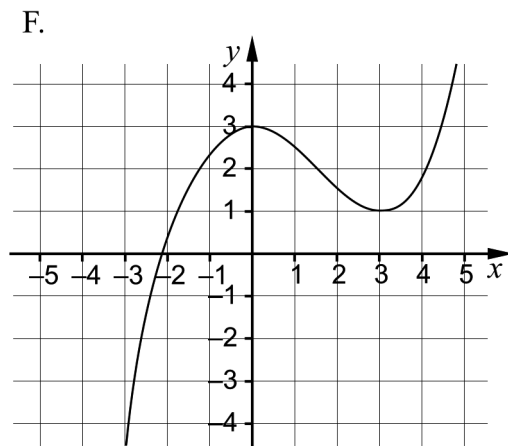
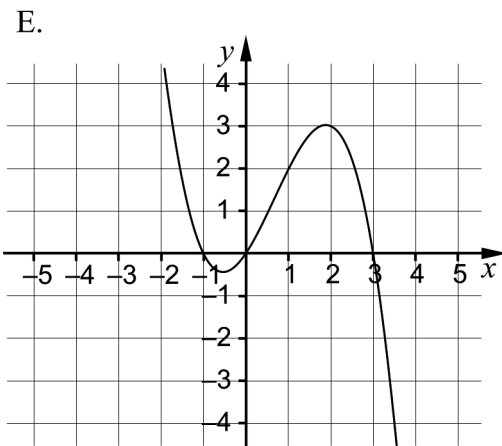
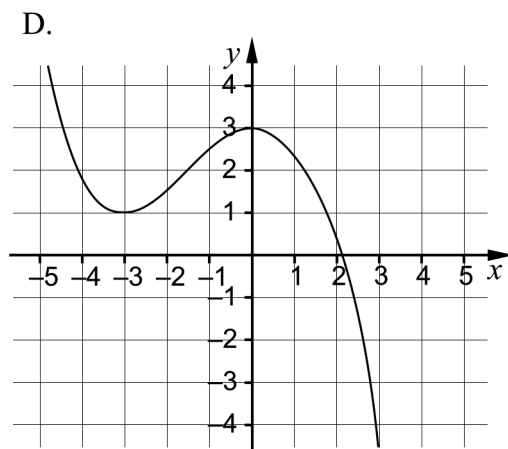
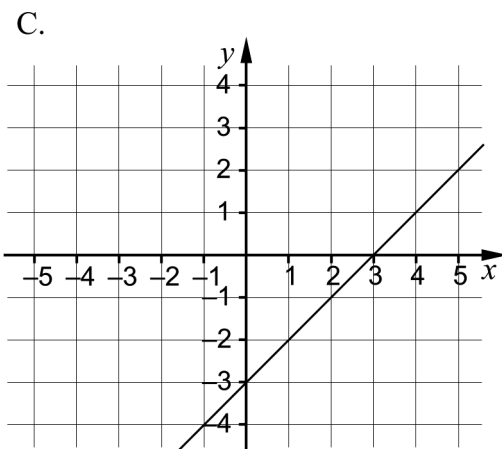
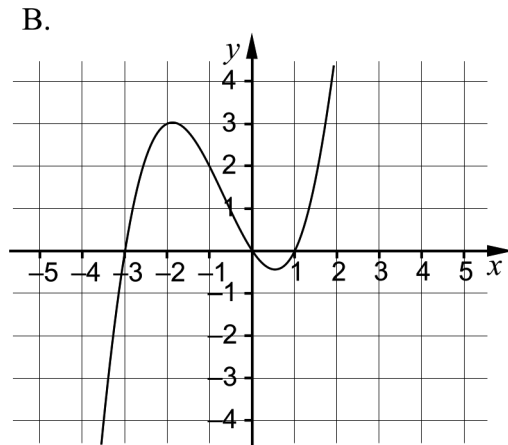
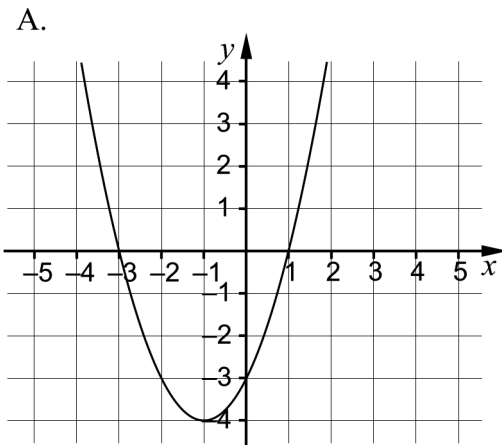
a) Find  $\arg z_1$  \_\_\_\_\_ (1/0/0)

b) Find  $\arg z_3$  \_\_\_\_\_ (0/1/0)

7. Chen is going to differentiate the function  $f$ . He sees that the function is a product. Chen differentiates the function and gets the correct answer  
 $f'(x) = 2x \cdot \sin x + x^2 \cdot \cos x$ .

Find the function  $f$ . \_\_\_\_\_ (0/1/0)

8. The figures A–F show the graphs of six different polynomial functions.



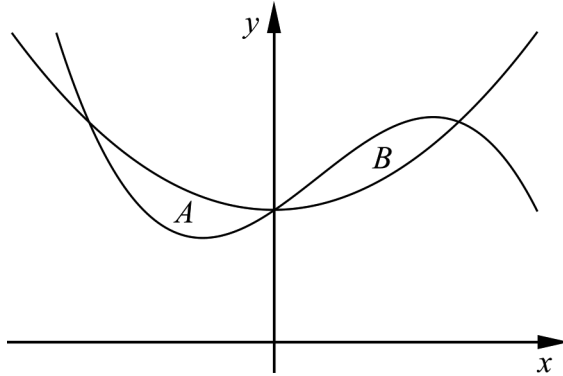
Two of the figures A–F show the graphs of polynomial functions that are divisible by  $x + 3$ . Which two?

\_\_\_\_\_ (0/0/1)

**Part C:** Digital resources are not allowed. Do your solutions on separate sheets of paper.

9. Show that  $\frac{\sin 2x}{2 \cos x} = \sin x$  for all  $x$  where the expressions are defined. (2/0/0)
10. Solve the equation  $\sin 3x = \frac{1}{2}$ . Give the answer in degrees. (2/1/0)
11. In the two complex numbers  $z_1 = a + ai$  and  $z_2 = (a+1) + (a-1)i$  the constant  $a$  is a real number and  $a > 0$ . Show that  $|z_1| < |z_2|$ . (0/2/0)
12. The equation  $x^2 + ax + b = 0$  has one root  $x = 1 + i\sqrt{3}$ . Determine the real constants  $a$  and  $b$ . (0/3/0)
13. One solution to the equation  $z^3 + 2z^2 + 5z + 10 = 0$  is  $z = -2$ . Determine the remaining solutions to the equation. (0/2/0)
14. Investigate how the number of solutions to the equation  $B \sin 2x = 5$  on the interval  $0 \leq x < 2\pi$  depends on the value of the constant  $B$ . Justify that the equation has the number of solutions you claim for the different values of  $B$ . (0/2/1)
15. Determine the constant  $a$  so that  $\int_2^4 \left( \frac{1}{x+2} + \frac{1}{x} \right) dx = \ln a$ . (0/1/1)
16. Solve the equation  $|z|^2 = 5z - 10i$ . (0/0/3)

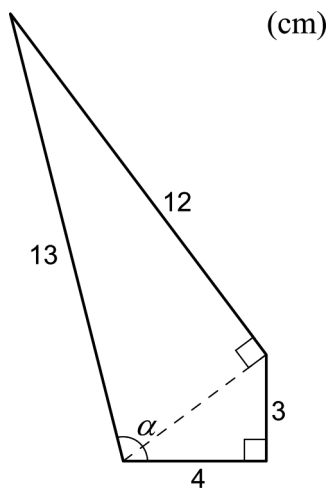
17. It holds for the functions  $f$  and  $g$  that  $f(x) = x^2 + 3$  and  $g(x) = -x^3 + x^2 + kx + 3$ , where  $k > 0$ . The graphs of the functions  $f$  and  $g$  enclose the regions  $A$  and  $B$ , see figure.



Show that the area of  $A$  is equal to the area of  $B$  regardless of the value of  $k$ .

(0/0/4)

18. The figure shows a quadrangle divided into two right-angled triangles.



One of the angles in the quadrangle is denoted  $\alpha$ . Determine  $\sin \alpha$ .

(0/0/2)