Part D	Problems 21–28 which require complete solutions.
Test time	120 minutes.
Resources	Digital resources, formula sheet and ruler.

The test consists of three written parts (Part B, Part C and Part D).
Together they give a total of 60 points consisting of
22 E-, 21 C- and 17 A-points.

Level requirements for test grades
E: 14 points
D: 23 points of which 7 points on at least C-level
C: 30 points of which 12 points on at least C-level
B: 39 points of which 5 points on A-level
A: 47 points of which 9 points on A-level

The number of points you can get for a complete solution is stated after each problem. You can also see what knowledge levels (E, C and A) you can show in each problem. For example (3/2/1) means that a correct solution gives 3 E-, 2 C- and 1 A-point.

For problems labelled *"Only answer is required"* you only have to give a short answer. For other problems you are required to present your solutions, explain and justify your train of thought and, where necessary, draw figures and show how you use your digital resources.

Write your name, date of birth and educational programme on all the sheets you hand in.

Name: ____________________

Date of birth: ____________________

Educational programme: ____________________

Part D: Digital resources are allowed. Write down your solutions on separate sheets of paper.

21. Determine the largest root of the equation $\sin x + \cos(3.6x) = 0$ on the interval $0° < x < 180°$
Write your answer to at least three significant figures. (2/0/0)

22. Rasmus studies the graphs of $y = 3\sin x$ and $y = 2\cos x$. He notices that the largest values are 3 and 2, respectively. He then concludes that the largest value of $y = 3\sin x + 2\cos x$ must be $3 + 2 = 5$

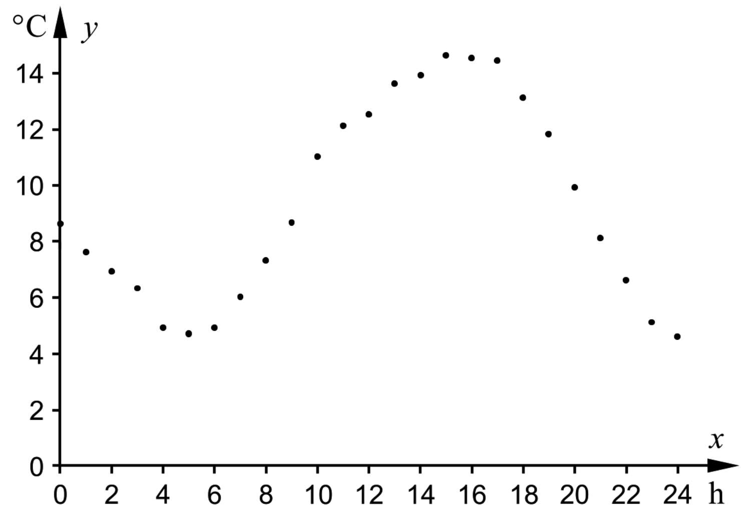
Rasmus checks this by drawing the graph of $y = 3\sin x + 2\cos x$ on his graphic calculator and then discovers that the largest value is less than 5

Use the graphs of $y = 3\sin x$ and $y = 2\cos x$ to explain why the largest value of $y = 3\sin x + 2\cos x$ is not 5 (1/1/0)

23. One of the SMHI weather observation stations is located in Torup. It measures the temperature once per hour.

If the daily (i.e. 24 h) mean temperature exceeds 10 °C for five consecutive days, the summer is considered to have started.

During the four days, April 20–23 2014, the daily mean temperature exceeded 10 °C in Torup. The diagram shows the temperatures measured on April 24.



According to a simplified model, the temperature during this 24 hour period can be described by the function

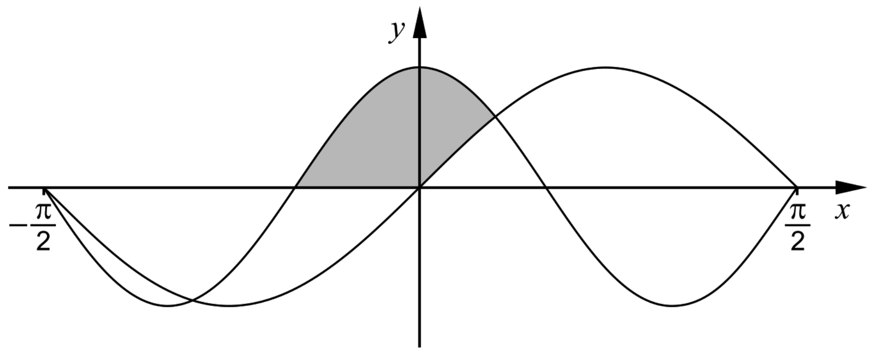
$$f(x) = -0.0079x^3 + 0.238x^2 - 1.43x + 8.2 \qquad 0 \le x \le 24$$

where $f(x)$ is the temperature in °C and x is the time in hours after 0:00

> Fact box: If the temperature is given by $f(x)$, the mean temperature of the time interval $a \le x \le b$ can be calculated the following way:
> $$\frac{1}{b-a}\int_a^b f(x)\,\mathrm{d}x$$

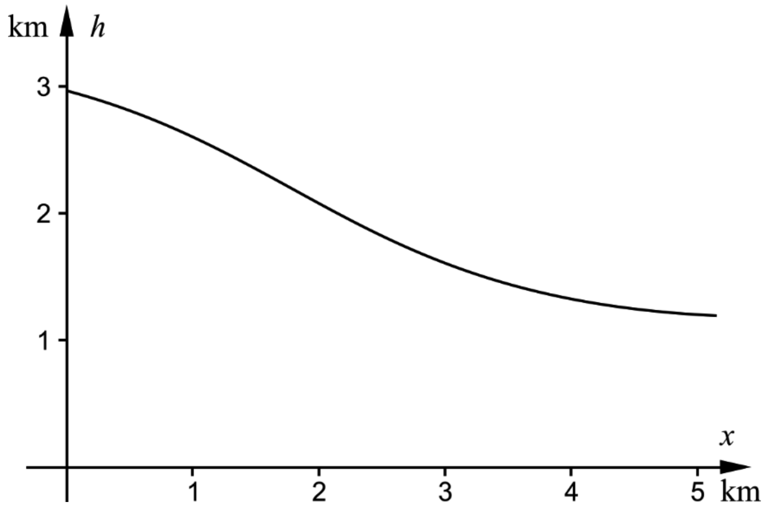
Decide whether the summer had started in Torup by determining the daily mean average temperature on April 24 by using the function. (2/0/0)

24. The figure shows a coordinate system where the curves $y = \cos 3x$ and $y = \sin 2x$ are drawn on the interval $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$



Calculate the area of the shaded region.
Write your answer to at least two significant figures. (1/3/0)

25. A company is building a cabin in the Alps and wants to know the slope of the hill. According to a simplified model, the shape of the hill can be described by the relation $h(x) = 4.1 - \frac{5 + 3e^x}{6 + e^x}$ where $h(x)$ is the height in km above sea level and x is the horizontal distance in km.



The company is building the cabin at a part of the hill that is 1.4 km above sea level. Calculate the slope of the hill where the cabin will be built. Answer to at least two significant figures. (0/2/0)

26. A company wants to control the life span of a certain kind of lamps. The time until a lamp is broken has turned out to be a random variable with a probability density function $f(x) = \dfrac{e^{-x/24}}{24}$, $x \geq 0$ where x is the time in months that the lamp is in use.

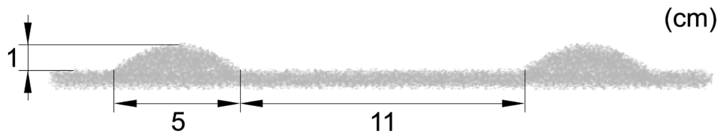
a) Determine the probability that a randomly chosen lamp breaks during the first 3 months of usage. (0/2/0)

b) Assume that three lamps are chosen at random. Determine the probability for all three lamps being unbroken after 6 months of usage. (0/0/2)

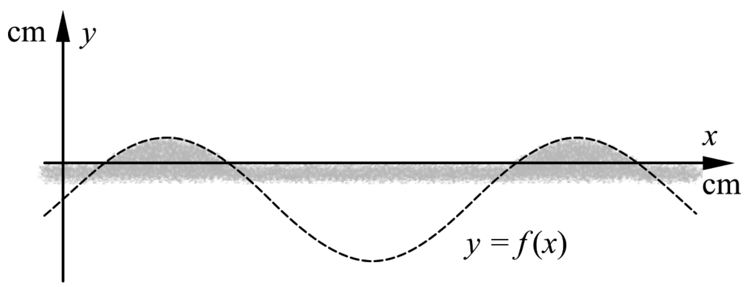
27. Investigate if the polynomial $p(x) = x^5 + a^4x^4 - x^3 + a^2x^2 + x + 1$ is divisible by $x - 1$ for any real value of the constant a. (0/0/2)

28. On the seabed near sandy beaches, periodic patterns of mounds are sometimes formed in the sand.

Assume that the height of a mound is 1 cm, the width is 5 cm and the distance between two mounds is 11 cm. See figure below.



According to a simplified model, each mound follows the top of a sine curve given by the function $f(x) = A\sin(kx) - d$ where A, k and d are positive constants. See figure below.



a) Determine the value of the constant k (0/1/0)

b) Determine the value of the constants A and d. (0/0/3)